

SOME SELECTED FEM ANALYSES OF MOTION OF A BULLET WITH IMPERFECTIONS WHILE INTERACTING WITH THE BARREL

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Abstract

The intended aim of studies on the modelling of a bullet interacting with a barrel is to generate a virtual, computer-based model, experimentally verified for some selected mating conditions, and to become able to predict – with strong probability – processes and phenomena that take place within a system with imperfections. In the case under consideration the problem of contact between the barrel and the bullet has been described with the calculation method based on the penalty function. Calculations were made using the so-called direct-integration procedure, colloquially called the „explicit integration”. Preliminary analysis has been carried out on the effect of the shift of the centre of mass of the bullet moving in the barrel. Simulation included the bullet displacement along the barrel’s axis, with the assumed positions of the centre of mass –5% and +5% as measured from the bottom and referred to the initial model (0%). Results of numerical calculations suggest the effect of shift of the bullet’s centre of mass, which - when exposed to complex states of loading - experiences oscillating movements of the bottom and the tip. Additionally, this analysis proves that there is a complex state of stress due to compression, torsion, and bending of the bullet while interacting with the inner part of the barrel. Numerical solutions may support the analytically approached solving of such mechanical system. They may also offer more precise determination of, e.g. initial and boundary conditions for the bullet’s free flight, which is of great significance to the accuracy of fire, and therefore, to the weapons effectiveness.

Keywords: barrel, bullet, simulation, movement modelling, imperfection

1. Introduction

One of meaningful issues in the field of numerical methods is physical modelling. This is a process of generating an ideal system with account taken of features that play the most important part [8]. Such being the case, a physical model is usually represented with a system of differential equations, which may take imperfections, e.g. geometric or material ones, into consideration. The modelling of such imperfections, and then the analysis of how they affect the system’s operation is undoubtedly one of the greatest challenges of the nowadays knowledge. The imperfections in question may include inaccuracies of the eccentric position of the centre of mass of a bullet, or displacements along the axis of rotation thereof.

The intended aim of studies on the modelling of a bullet interacting with a barrel is to generate a virtual, computer-based model, experimentally verified for some selected mating conditions, and to become able to predict – with strong probability – processes and phenomena that take place within a system with imperfections [6].

2. FEM modelling

2.1. Energy equation

The stationary functional J_1 has been used to gain the basic FEM system of equations. This functional represents the sum of kinetic energy, internal energy (body strains) and potential energy of external load upon the body [8], and has been expressed in the following way:

$$J_1 = \int_{t_p}^{t_k} \left[\frac{1}{2} \dot{q}^T M \dot{q} - \frac{1}{2} q^T K q + Q^T q \right] d\tau, \quad (1)$$

where:

M - global mass matrix (consistent matrix),

K - global stiffness matrix,

q - vector of generalised displacements,

Q - a given vector of generalised exciting forces (these are non-potential and non-dissipative forces), forced excitations.

Finding an approximate solution of the boundary value problem with uniform conditions for a differential equation consists in looking for the minimum of the stationary functional J_1 , which is a necessary condition of the optimality. With the Hamilton's principle engaged we get [2]:

$$\delta J_1 = 0, \quad (2)$$

Assuming that prepared displacements equal zero for t_p and t_k , the following is arrived at:

$$\delta q(t_p) = 0 \text{ and } \delta q(t_k) = 0, \quad (3)$$

The condition of stationarity of the functional J_1 in the Hamilton's principle takes the following form [2]:

$$M\ddot{q} + Kq = Q, \quad (4)$$

While studying dynamic responses of structural systems, considerations on the dissipation of energy prove of great significance and complexity. When body displacement changes with time, two groups of additional forces appear [1, 15]. The first group results from the viscotic resistance of the medium, within which the bodies move, whereas the second one – from the internal friction. The in this way generated damping forces are twofold in nature [1]. With such assumptions made, damping forces should be taken into account in the equation of motion (4), by adding of $C\dot{q}$. Hence, the equation of motion should be expressed in the following way:

$$M\ddot{q} + C\dot{q} + Kq = Q. \quad (5)$$

where:

$C = \alpha_1 M + \alpha_2 K$ - damping matrix,

α_1, α_2 - coefficients found on the basis of modal analyses [4].

A model of the barrel-bullet system, given consideration in this paper, can be described with a system N of differential equations presented in the form of a matrix consistent with eq. (5). The elements of the system join with each other in the nodes. Dependences found for particular elements need to be extended upon the whole domain under analysis. This requires two conditions to be satisfied:

- condition of consistency (functions to be found should have continuous derivatives of the $m-1$ order on inter-element boundaries),
- condition of completeness (functions searched for, inside the elements, should have the m -th continuous derivatives).

The above-mentioned conditions, when satisfied, ensure convergence of the solution approximated in the problem described with the functional that comprises derivatives up to the m -th degree inclusive.

2.2. General equations

Conditions of equilibrium of the body for time intervals t and $t + \Delta t$ have been assumed to satisfy the needs of computer-based simulation under severe, high-speed, and dynamic loads. The problem of non-linear dynamics has been described with a basic system of equations of motion. It is expressed in the following way [8, 6]:

- geometric equations

$$\varepsilon_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}), \quad (6)$$

- constitutive equations

$$\sigma_{kl} = C_{klmn} \varepsilon_{mn}, \quad (7)$$

- displacement boundary conditions

$$u_k = \hat{u}_k, \quad (8)$$

- initial conditions

$$u_k = \hat{u}_k^o, \quad \dot{u}_k = \hat{\dot{u}}_k, \quad (9)$$

where:

$u_{k,l}, u_{l,k}$ - components of displacements of any element inside a deformable body,

$\varepsilon_{kl}, \varepsilon_{mn}$ - components of strain tensor,

C_{klmn} - matrix of compliance (constitutive),

σ_{kl} - components of stress tensor,

u_k - components of vector of displacement on the body's edge,

\dot{u}_k - components of vector of velocity on the body's edge,

\hat{u}_k - boundary condition of displacement on the body's edge,

\hat{u}_k^o - initial condition of displacement of the body's edge,

$\hat{\dot{u}}_k$ - boundary condition of velocity of the body's edge,

$\hat{\dot{u}}_k^o$ - boundary condition of velocity of the body's edge.

2.3. Constitutive equations of the material

For the elastic range, the deformation process has been described with the stress deviator:

$$D_\sigma = 2G D_\varepsilon, \quad (10)$$

whereas the isotropic component of the stress tensor (axiator) is expressed with the following relationship:

$$A_{\sigma} = 3K A_{\varepsilon}, \quad (11)$$

where:

- G - modulus of elasticity in shear,
- K - modulus of volume elasticity,
- D_{ε} - strain deviator,
- A_{ε} - isotropic component of the strain tensor (axiator).

The process of the material plasticization takes place at the moment of transition from the elastic state to the plastic one. The yield point determines limits of the state of plasticization. In the case of the barrel-bullet system, the transition to the plastic state occurs, e.g. in the bullet's jacket.

The isotropic hardening of the material manifests itself in that the yield stress is independent of the loading technique; The plastic flow area keeps extending with its form preserved. The extension of the area of plastic flow depends on the strain hardening parameter κ . The equivalent plastic strain ε is assumed measure of this parameter [4, 8]:

$$\kappa = \int_0^{\varepsilon} d\varepsilon \quad (12)$$

In the range of plastic strains the yield point is modified, according to [4, 8]:

$$\sigma_y = \sigma_o + \beta \frac{3}{2} E_p \varepsilon_{eff}^p, \quad (13)$$

where:

- β - strain hardening parameter,
- E_t - tangential modulus,
- E - Young's modulus for linear range,
- E_p - linear strain hardening.

Expression that describes relationships for the plastic strain takes the following form:

$$E_p = \frac{2}{3} \frac{E_t E}{E - E_t}, \quad (14)$$

whereas the principle of flow looks like this:

$$\varepsilon_{eff}^p = \int_0^t \left(\frac{2}{3} \dot{\varepsilon}_{kl}^p \dot{\varepsilon}_{kl}^p \right)^{\frac{1}{2}} dt. \quad (15)$$

2.4. The FEM model of the barrel-bullet system

The process of generating the FEM model of a barrel with grooves starts with the forming of „a slice” made of solid finite elements. Finite elements in the form of „slices” are formed by means of joining node meshes that represent adjacent cross-sections. Dimensions of finite elements used to generate a model of a barrel result from the depth of grooves in the barrel. The 8-node elements used in the FEM model show the shortest to longest edge ratio 1:5. This ratio

ensures stability of the solution and reduces the number of finite elements applied. Fig. 1a shows the barrel-bullet system, whereas Fig. 1b – the cartridge chamber, partitioned into finite elements. The FEM modelling of the cartridge chamber is also relatively simple. In this subdomain the FEM partition is sparse (Fig. 1b).

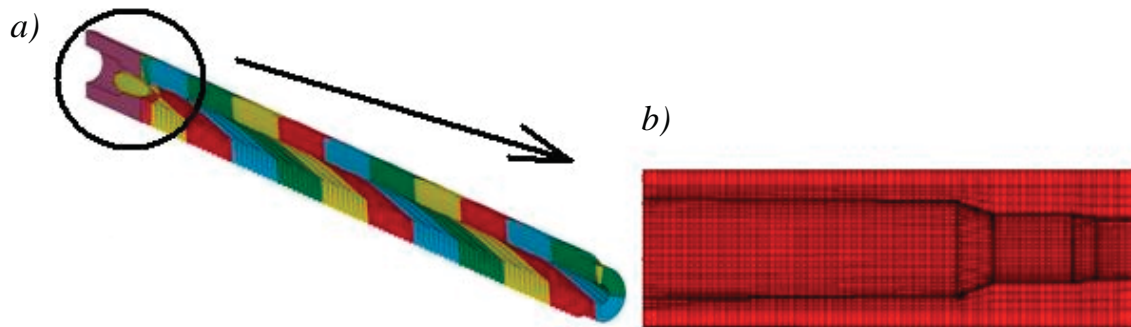


Fig. 1. FEM model of the grooved barrel-bullet system, and the cartridge chamber a) the barrel-bullet system b) the cartridge-chamber model, partitioned into elements

In the case under consideration, the bullet model with twisted FEM mesh (Fig. 2) was used to match geometry of the bullet to that of the barrel's grooves. Each layer of finite elements was rotated against the one going before by the angle of 50. Fig. 2 shows a discrete model of the bullet.

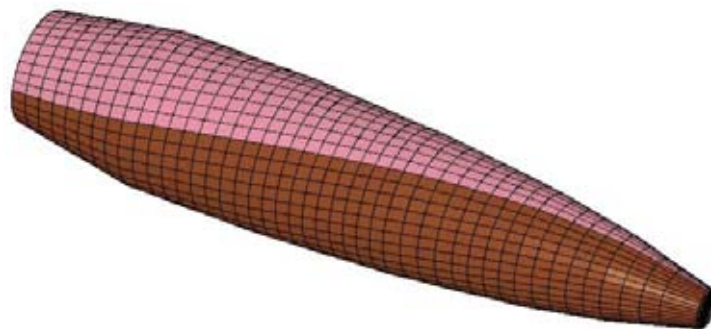


Fig. 2. FEM model of bullet under analysis, with mesh matched to a barrel with grooves

In the numerical barrel-bullet system, in the space behind the bullet, i.e. behind the zone where the barrel and the bullet interact, high-speed changes in pressure take place. The region of pressure impact in the barrel extends as the bullet approaches the muzzle. The case has been explained in detail in [2, 13, 14].

The barrel loading is carried out in this model at a uniform rate using sectors (segments) (Fig. 3a) connected in ring-shaped regions (Fig. 3b), joined together after the bullet's relocation (Fig. 3c). Sectors represent areas, formed of the finite-element wall affected by combustion-induced gas.

The bullet's motion is the effect of axial-symmetric pressure upon the model. In the problem under modelling the pressure loading took place on the rear section of the bullet, i.e. on its base and rear chamfered portion, see [2, 13, 14].

The change of pressure results with the combustion of the propellant explosive (powder), and then exhaust gases decompression. Changes of exhaust gases pressure with time have been shown in [2, 13, 14]. The course of pressure against time of propellant-charge combustion within some variable closed volume was gained experimentally. The experiment was carried out on a test stand described in [9]. The objective was to measure bullet's velocity at the distance $l = 2.5$ m from the muzzle. Measurements taken in the course of testing work showed $V_{2.5} = 901$ m/s. The numerical-analysis effected result was $V = 844$ m/s.

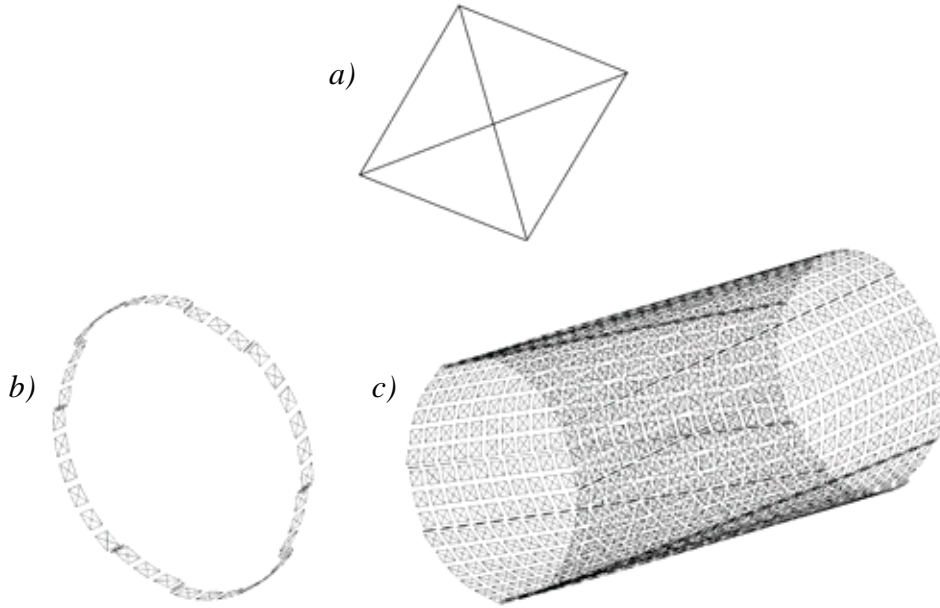


Fig. 3. Load implementation in the barrel, in the space behind the bullet: a) sectors (segments) upon finite element, which form the inner surface of the barrel, b) single region composed of ring-shaped segments, and c) connected regions that carry time – and space – dependent loading into effect

2.5 Algorithm of contact based on the penalty function

In the case under consideration the problem of contact between the barrel and the bullet has been described with the calculation method based on the penalty function [2, 4]. This function can be applied to normal displacements in the displacement-based approach; to normal velocities defined in the velocity-based approach; and to normal displacements in the velocity-based approach, the latter being the most often form of application. In the penalty function method the normal contact force is expressed with the following equation:

$$F_{nij} = \zeta u_{nji} H(-u_{nji}). \quad (16)$$

where:

$H(\cdot)$ - Heaviside step function,
 $\zeta = 1/\kappa$, κ - coefficient of the penalty function.

Conditions of contact are checked on the grounds of $\mathbf{B}u \geq \gamma$, where \mathbf{B} is matrix that describes boundary conditions kinematics, γ is vector of initial gap.

In the course of numerical application of this method an imaginary (fictitious) energetic term is added in the form of the penalty function [2, 4]: $\pi = 1/2 u^T \mathbf{K} u - u^T f + \kappa [(\mathbf{B}u - \gamma)^T (\mathbf{B}u - \gamma)]$. In terms of physical interpretation of the penalty function parameter, operation thereof should be interpreted as an imaginary (fictitious) elastic element that appears between two nodes in contact. Value of this parameter [2, 4] is found on the basis of accuracy of a computing machine, number of unknowns, and the least stiffness of elements in contact at the moment.

3. Effect of axial shift of the bullet's centre of mass upon its motion in the barrel

Preliminary analysis has been carried out on the effect of the shift of the centre of mass of the bullet moving in the barrel. Simulation included the bullet displacement along the barrel's axis, with the assumed positions of the centre of mass -5% and $+5\%$ as measured from the bottom and referred to the initial model (0%).

Fig. 4ab illustrates the effect of initial interaction between the grooves of the barrel and the pressure loading for three instances of axial shift of the centre of mass: +5%, 0%, -5%. Results of numerical calculations shown in Figs 4ab suggest the effect of shift of the bullet's centre of mass, which – when exposed to complex states of loading – experiences oscillating movements of the bottom and the tip. The point at the tip of the bullet experiences the greatest displacement for the shift -5% and +5%, whereas the lowest value occurs for the bullet in the initial model (0%). This proves that there is a complex state of stress due to compression, torsion, and bending of the bullet while interacting with the inner part of the barrel.

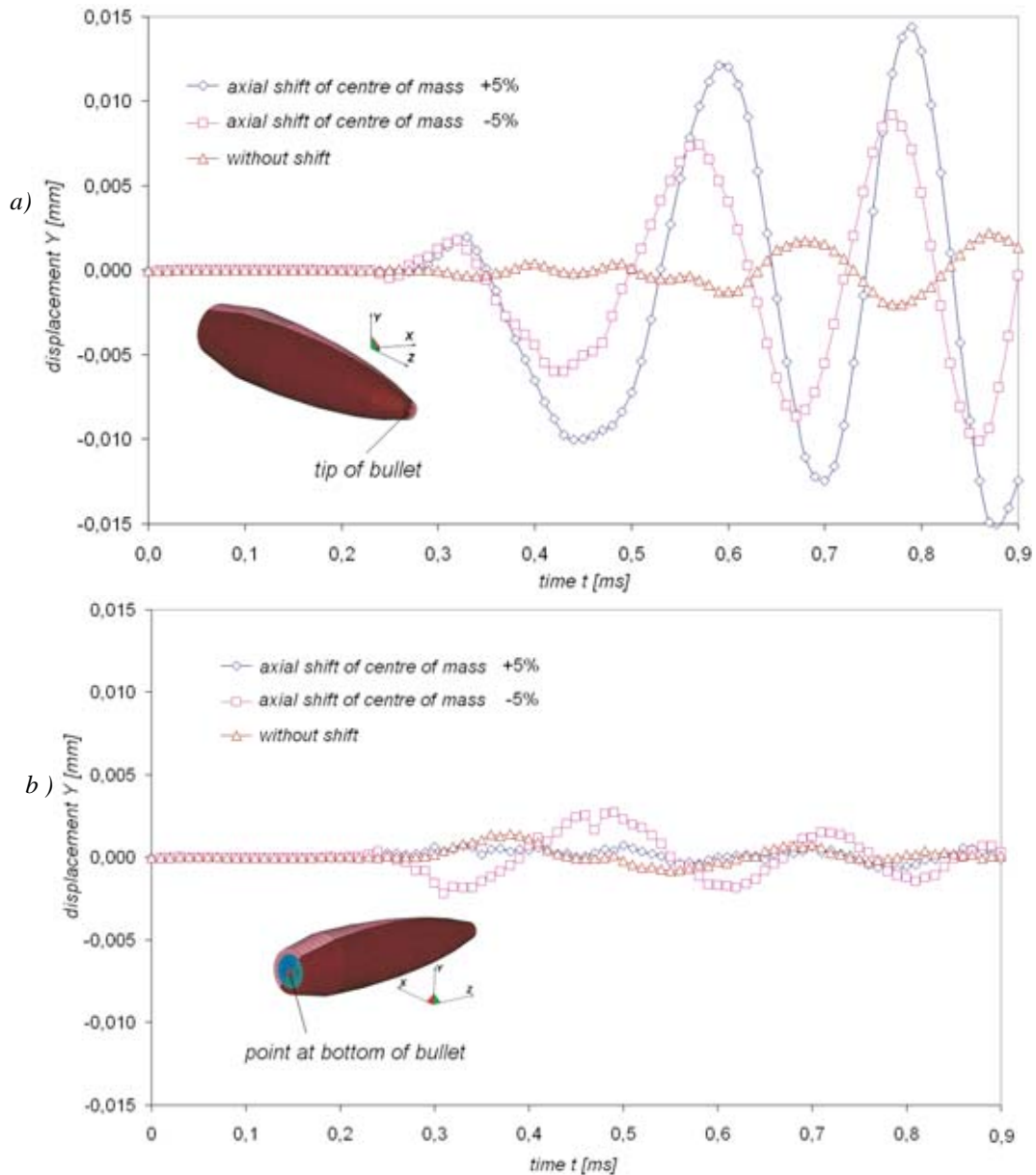


Fig. 4. Behaviour of the bullet in the barrel for different positions of the centre of mass – +5%, 0%, -5% : a) the tip of the bullet b) the point at the bottom of the bullet

4. Conclusions

The above-considered theoretical models may be applied to preliminary analyses of the interaction between the bullet and the barrel with numerical simulations engaged. They may prove helpful in determining this interaction as soon as at the design stage, and deliver data to determine, e.g. the state of stress in the course of the bodies' interaction. A numerical solution may prove

helpful in predicting, with strong probability, behaviour of the bullet while interacting with the barrel, since it provides analysis of different variants, e.g. of the change in the position of the centre of mass. Such being the case, numerical solutions may supplement the analytically approached solving of this mechanical system. They may also offer more precise determination of, e.g. initial and boundary conditions for the bullet's free flight, which is of great significance to the accuracy of fire, and therefore, to the weapons effectiveness.

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